Logs and Exponentials: Properties of Logs

Recall: $y = b^x \leftrightarrow log_b y = x$

Common properties that result from this (these are true for any base):

$log_5 0$ = undefined

$log_5 0 = x \rightarrow 5^x = 0 \rightarrow$ can't happen

$log_5 1$ = 0

$log_5 1 = x \rightarrow 5^x = 1 \rightarrow$ x = 0

$log_5 5$ = 1

$log_5 5 = x \rightarrow 5^x = 5 \rightarrow$ x = 1

Other Properties of Logarithms:

1. Product Rule = "The log of a product is the sum of the logs"

$log_b X \cdot Y = log_b X + log_b Y$

2. Quotient Rule = "The log of a quotient is the difference of the logs"

$$log_b \frac{X}{Y} = log_b X - log_b Y$$

3. Power Rule

$log_b X^a = a \cdot log_b X$

Ex: Expand to individual logs.

$log_b \frac{x}{z} = log_b x - log_b z$ (quotient rule)

$log_b x^3 y = log_b x^3 + log_b y = 3log_b x + log_b y$ (product rule, then power rule)

Ex: Put together as a single log.

$2log_b x + log_b y - 5log_b z = log_b x^2 + log_b y - log_b z^5 = log_b x^2 y - log_b z^5 = log_b \frac{x^2 y}{z^5}$

Ex: Suppose $log_b W = 4, log_b R = 3,\ find\ log_b \frac{R}{W^2}$

$log_b \frac{R}{W^2} = log_b R - log_b W^2 = log_b R - 2log_b W = 3 - 2 \times 4 = 3 - 8 = -5$